# Four mathematical formulations of the Optimal Power Flow problem and their impacts on the performance of solution methods

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## Introduction

The Optimal Power Flow (OPF) problem is computed to obtain the optimal operational state of the electrical power system instead of steady state while satisfying system constraints and control limits. We study how the different mathematical formulations of the OPF problem affect the performance of the solution method while keeping the same physical formulation. To identify the formulation that results in the best convergence characteristics for the solution method, we apply the Interior Point Method (IPM) to all four mathematical formulations of the OPF problem.

## Physical formulation of the OPF problem

The objective is to minimize the total cost for the active power generation:

$$\min_{\mathbf{u}} \; f(\mathbf{x},\mathbf{u}) = \sum_{i=1}^{N_g} \left(C_i^0 + C_i^1 P_i^g + C_i^2 (P_i^g)^2\right) \tag{1}$$

subject to equality constraints $g(\mathbf{x},\mathbf{u})$ as power flow equations:

$$g_i(\mathbf{x},\mathbf{u}) = S_i - V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* = 0 \qquad \forall i \in 1,...,N \tag{2}$$

and inequality constraints $h(\mathbf{x},\mathbf{u})$ as squared branch flow limits for the apparent power:

$$h_{ij}(\mathbf{x},\mathbf{u}) = \begin{bmatrix} |S_{ij}^f(\mathbf{x},\mathbf{u})|^2 \\ |S_{ij}^t(\mathbf{x},\mathbf{u})|^2 \end{bmatrix} \leq \begin{bmatrix} (S_{ij}^{\max})^2 \\ (S_{ij}^{\max})^2 \end{bmatrix} \tag{3}$$

and limits for control variables $\mathbf{u}$:

$$\mathbf{u}^{\min} \leq \mathbf{u} \leq \mathbf{u}^{\max} \tag{4}$$

where

$$\mathbf{u} = \left[\delta_1, .., \delta_N, |V_1|, .., |V_N|, P_1^g, .., P_{N_g}^g, Q_1^g, .., Q_{N_g}^g\right]^T \tag{5}$$

or

$$\mathbf{u} = \left[V_1^m, .., V_N^m, V_1^r, .., V_N^r, P_1^g, .., P_{N_g}^g, Q_1^g, .., Q_{N_g}^g\right]^T \tag{6}$$

## Mathematical formulations of OPF problem

The equality constraints (2) can be reformulated in four different ways using the power and current balance equations in polar and Cartesian coordinates [1] as shown in (7)-(10):
Power balance equations in polar coordinates (pp):

$$g_i(\mathbf{x},\mathbf{u}) = \begin{bmatrix} P_i^{sp} - \sum_{k=1}^N |V_i||V_k|\left(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}\right) \\ Q_i^{sp} - \sum_{k=1}^N |V_i||V_k|\left(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}\right) \end{bmatrix} \tag{7}$$

Power balance equations in Cartesian coordinates (pc):

$$g_i(\mathbf{x},\mathbf{u}) = \begin{bmatrix} P_i^{sp} - \sum_{k=1}^N \left(V_i^r(G_{ik}V_k^r - B_{ik}V_k^m) + V_i^m(B_{ik}V_k^r + G_{ik}V_k^m)\right) \\ Q_i^{sp} - \sum_{k=1}^N \left(V_i^m(G_{ik}V_k^r - B_{ik}V_k^m) - V_i^r(B_{ik}V_k^r + G_{ik}V_k^m)\right) \end{bmatrix} \tag{8}$$

Current balance equations in polar coordinates (cp):

$$g_i(\mathbf{x},\mathbf{u}) = \begin{bmatrix} \frac{P_i^{sp}\cos\delta_i + Q_i^{sp}\sin\delta_i}{|V_i|} - \sum_{k=1}^N |V_k|\left(G_{ik}\cos\delta_k - B_{ik}\sin\delta_k\right) \\ \frac{P_i^{sp}\sin\delta_i - Q_i^{sp}\cos\delta_i}{|V_i|} - \sum_{k=1}^N |V_k|\left(G_{ik}\sin\delta_k + B_{ik}\cos\delta_k\right) \end{bmatrix} \tag{9}$$

Current balance equations in Cartesian coordinates (cc):

$$g_i(\mathbf{x},\mathbf{u}) = \begin{bmatrix} \frac{P_i^{sp}V_i^r + Q_i^{sp}V_i^m}{(V_i^r)^2 + (V_i^m)^2} - \sum_{k=1}^N (G_{ik}V_k^r - B_{ik}V_k^m) \\ \frac{P_i^{sp}V_i^m - Q_i^{sp}V_i^r}{(V_i^r)^2 + (V_i^m)^2} - \sum_{k=1}^N (G_{ik}V_k^m + B_{ik}V_k^r) \end{bmatrix} \tag{10}$$

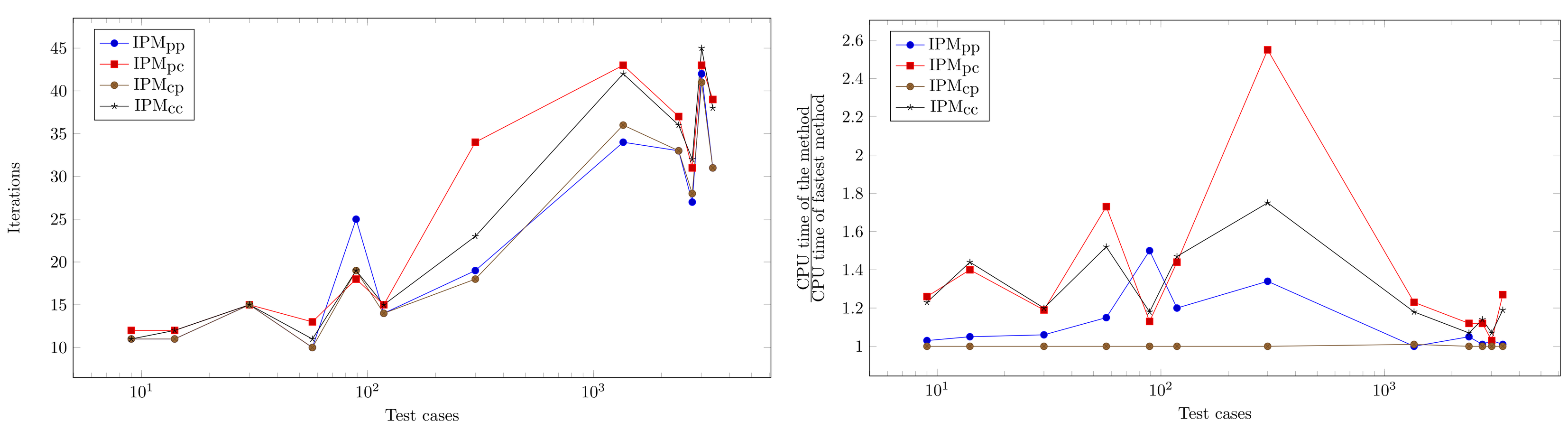
Combining (1) and (3)-(4) with the one of (7)-(10) and a coordinate choice in (5)-(6), we can obtain four different mathematical formulations for a single physical formulation of the OPF problem.

## Interior Point Methods

We use one of the deterministic hybrid methods that combines the IPM with Lagrangian relaxation and Newton's method. For more detailed information of all four variants of the IPM, we refer to [2, 3, 4]:

- $\text{IPM}_{\text{pp}}$ using power balance equations in polar coordinates
- $\text{IPM}_{\text{pc}}$ using power balance equations in Cartesian coordinates
- $\text{IPM}_{\text{cp}}$ using current balance equations in polar coordinates
- $\text{IPM}_{\text{cc}}$ using current balance equations in Cartesian coordinates

## Numerical results




*(a) Number of iterations* *(b) Ratio for CPU times*

*Figure 1: Comparison of four variants of the IPM on different test cases*

## Conclusion

Results show that the performance of the OPF solution method is not only dependent upon the choice of the solution method itself, but also upon the exact mathematical formulation used to specify the OPF problem.

## References

[1] B. Sereeter, K. Vuik, and C. Witteveen, "Newton power flow methods for unbalanced three-phase distribution networks," *Energies*, vol. 10, no. 10, p. 1658, 2017.

[2] H. Wang, C. E. Murillo-Sanchez, R. D. Zimmerman, and R. J. Thomas, "On computational issues of market-based optimal power flow," *IEEE Transactions on Power Systems*, vol. 22, no. 3, pp. 1185–1193, 2007.

[3] B. Sereeter and R. D. Zimmerman, "*Addendum* to AC power flows and their derivatives using complex matrix notation: Nodal Current Balance," March 2018. Available: http://www.pserc.cornell.edu/matpower/TN3-More-OPF-Derivatives.pdf.

[4] B. Sereeter and R. D. Zimmerman, "AC power flows and their derivatives using complex matrix notation and Cartesian coordinate voltages," March 2018. Available: http://www.pserc.cornell.edu/matpower/TN4-OPF-Derivatives-Cartesian.pdf.